

# Letters

## Comment on "A Field Theoretical Derivation of TLM"

S. Hein

Contrary to what is claimed in the above cited paper,<sup>1</sup> former mathematical derivations of the TLM method directly from Maxwell's equations (meeting all requirements of rigor, but less complicated than the paper) have been given and can be found, e.g., in [1] and [2].

*Author's Reply by Michael Krumpholtz and Peter Russer*

We appreciate the comments on our paper. Apparently, the statement in our paper he is referring to "However a mathematical derivation of the TLM method directly from Maxwell's equations has not yet been given..." needs some further explanation. By direct derivation we mean a derivation that uses only Maxwell's equation and no further physical assumptions such as energy conservation of the wave amplitudes or averaging of field components. In Hein's paper [2] averaging is applied and in the paper by Chen *et al.* [1] the derivation is based not only on Maxwell's equations but also on the additional assumption of energy conservation.

In our method of moments approach the only assumptions one has to apply is to specify the expansion functions and the test functions. Our derivation is solely based on Maxwell's equations and the method of moments approximation. Thus the physical basis and the mathematical treatment are clearly separated. By this rigorous approach for example the problem of correct mapping between the field variables and the wave amplitudes could be solved.

### REFERENCES

- [1] Z. Chen, M. M. Ney, and W. J. R. Hoefer, "A new finite-difference time-domain formulation and its equivalence with the TLM symmetrical condensed node," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 2160–2169, 1991.
- [2] S. Hein, "Consistent finite-difference modelling of Maxwell's equations with lossy symmetrical condensed TLM node," in *Int. J. Numerical Modelling*, vol. 6, pp. 207–220, 1993.

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<sup>1</sup>M. Krumpholtz and P. Russer, *IEEE Trans. Microwave Theory Tech.*, vol. 42, pp. 1660–1668, Sept. 1994.

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## Comments on "Coaxial Probe Modeling in Waveguides and Cavities"

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In the above paper,<sup>1</sup> Liang *et al.* presented an excellent analysis of a coaxial probe in waveguides and cavities. The compact dyadic Green's function technique was applied there to calculate the electric field. However, there are some typographical errors of the dyadic Green's function components in that paper which will mislead further investigations. These errors found during the procedure of reproducing their results are listed below as corrections.

In (A-5) and (A-6) of the paper, there is a surplus minus sign. The form of (A-5) and (A-6) is

$$A_{mn} = \frac{4\epsilon}{ab\epsilon_{0m}} \frac{\sin[k_x(x' - a/2)] \cos(k_y y') \cosh[\gamma_{mn}(z' - l_2)]}{\gamma_{mn} \sinh[\gamma_{mn}(l_1 + l_2)]} \quad (\text{A-5})$$

$$B_{mn} = \frac{4\epsilon}{ab\epsilon_{0m}} \frac{\sin[k_x(x' - a/2)] \cos(k_y y') \cosh[\gamma_{mn}(z' + l_1)]}{\gamma_{mn} \sinh[\gamma_{mn}(l_1 + l_2)]} \quad (\text{A-6})$$

In their (A-10), (p. 2178) there is a minus sign missing. The correct form should be given by

$$\frac{\sinh \gamma_{mn}(z - l_2) \sinh \gamma_{mn}(z' + l_1)}{\gamma_{mn} \sinh \gamma_{mn}(l_1 + l_2)} \rightarrow -\frac{e^{-k_c(|z - z'|)}}{2k_c} \quad (\text{A-10})$$

There is a typographical error  $z^P$  for the case  $m = 0$  in (A-12). The correct form is

$$S_{m,\infty}^{Gayy} = \sum_{n=1}^{\infty} \sin[k_x(x - a/2)] \sin[k_x(x' - a/2)] \left( -\frac{e^{-k_c(|z - z'|)}}{2k_c} \right) \\ = -\frac{a}{4\pi} \Re e \left\{ \ln \left[ \frac{1 - e^{-j\frac{\pi}{a}(|z + x' + a| + |z - z'|)}}{1 - e^{-j\frac{\pi}{a}(|x - x'| + |z - z'|)}} \right] \right\}, \quad m = 0. \quad (\text{A-12})$$

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<sup>1</sup>J.-F. Liang, H. C. Chang, and K. A. Zaki, *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 2172–2180, 1992.

In their (A-16) there is a typing error  $\mu$  that should be replaced by  $\epsilon$  and a related error due to (A-5), so that

$$\begin{aligned} G_{F_{xx}}(x, y, z; x', y', z') \\ = \frac{4\epsilon}{ab} \sum_{m=0}^{\infty} \frac{\cos(k_y y) \cos(k_y y')}{\epsilon_{0m}} \\ \cdot \left\{ \sum_{n=1}^{\infty} \sin[k_x(x - a/2)] \right. \\ \cdot \sin[k_x(x' - a/2)] (S_{mn}^{G_{F_{xx}}} - S_{mn,\infty}^{G_{F_{xx}}}) + S_{m,\infty}^{G_{F_{xx}}} \}. \end{aligned} \quad (\text{A-16})$$

Similarly in their (A-19) there is also a typing error  $\mu$ . The corrected form gives

$$\begin{aligned} G_{F_{zz}}(x, y, z; x', y', z') \\ = -\frac{4\epsilon}{ab} \sum_{m=0}^{\infty} \frac{\cos(k_y y) \cos(k_y y')}{\epsilon_{0m}} \\ \cdot \left\{ \sum_{n=1}^{\infty} \cos[k_x(x - a/2)] \right. \\ \cdot \cos[k_x(x' - a/2)] (S_{mn}^{G_{F_{zz}}} - S_{mn,\infty}^{G_{F_{zz}}}) + S_{m,\infty}^{G_{F_{zz}}} \} \end{aligned} \quad (\text{A-20})$$

where

$$S_{mn,\infty}^{G_{F_{zz}}} = -\frac{e^{-k_c(|z-z'|)}}{2k_c}. \quad (\text{A-22})$$

There are not only some typographical errors such as the braces, i.e., { and } and the brackets, i.e., ( and ), but also a critical error in  $F$  of the logarithm function  $\ln(F)$ , which is caused by the sign change. According to the summation formulas (A-6) given by Leviatan *et al.* [1],  $e^{j(\bar{x} \pm j \bar{y})}$  was used in the summations of the geometric series. This sign change in their (A-12) does not affect the results after the real value is taken because of (see [1])

$$\begin{aligned} \Re e \left\{ \ln \left[ \frac{1 - e^{-j \frac{\pi}{a}(|x+x'+a|+j|z-z'|)}}{1 - e^{-j \frac{\pi}{a}(|x-x'|+j|z-z'|)}} \right] \right\} \\ = \Re e \left\{ \ln \left[ \frac{1 - e^{j \frac{\pi}{a}(|x+x'+a|+j|z-z'|)}}{1 - e^{j \frac{\pi}{a}(|x-x'|+j|z-z'|)}} \right] \right\} \end{aligned}$$

because the factor  $\frac{\pi}{a}|z-z'|$  can be cancelled in this case. However, this factor cannot be cancelled in (A-23) so that the sign change does

affect (A-23). The correct form can be written as

$$\begin{aligned} S_{m,\infty}^{G_{F_{zz}}} &= \sum_{n=1}^{\infty} \cos[k_x(x - a/2)] \cos[k_x(x' - a/2)] \left( -\frac{e^{-k_c(|z-z'|)}}{2k_c} \right) \\ &= \begin{cases} \frac{a}{4\pi} \Re e \left\{ \ln \left[ 1 - e^{j \frac{\pi}{a}(|x+x'+a|+j|z-z'|)} \right] \right. \\ \quad \left. + \ln \left[ 1 - e^{j \frac{\pi}{a}(|x-x'|+j|z-z'|)} \right] \right\}, & m = 0, \\ + \frac{1}{4k_y} e^{-k_y|z-z'|} - \frac{a}{4\pi} \sum_{n=-\infty}^{\infty} \\ \quad \cdot \{ K_0[k_y \sqrt{(x-x'+2na)^2 + (z-z')^2}] \\ \quad + K_0[k_y \sqrt{(x+x'+a+2na)^2 + (z-z')^2}] \}, & m \neq 0. \end{cases} \end{aligned} \quad (\text{A-23})$$

*Author's Reply by Ji-Fuh Liang, H. C. Chang, and K. A. Zaki*

The authors welcome the comments for the typographical errors in our paper. We also found errors in (A12) and the correct form should be read as

$$\begin{aligned} S_{m,\infty}^{G_{F_{zz}}} Ayy &= \sum_{n=1}^{\infty} \sin k_x(x' - a/2) \sin k_x(x - a/2) \\ &\cdot \left( \frac{-e^{-k_c|z-z'|}}{2k_c} \right) \\ &= -\frac{a}{4\pi} \Re e \left\{ \ln \frac{1 - e^{j \frac{\pi}{a}(|x+x'+a|+j|z-z'|)}}{1 - e^{j \frac{\pi}{a}(|x-x'|+j|z-z'|)}} \right\}, \\ &m = 0 \\ &= -\frac{a}{4\pi} \sum_{n=-\infty}^{\infty} \\ &\cdot \{ K_0(k_y \sqrt{(x-x'+2na)^2 + (z-z')^2}) \\ &\quad - K_0(k_y \sqrt{(x+x'+a+2na)^2 + (z-z')^2}) \} \\ &m \neq 0. \end{aligned} \quad (\text{A12})$$

## REFERENCES

- [1] Y. Leviatan, P. G. Li, A. T. Adams, and J. Perini, "Single-post inductive obstacle in rectangular waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 806-812, 1983.

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